Tutorial 7:

3.2-6.

(a) $PR_t = K - I_t = K - 103$, $\quad PR_{t+1} = K - I_{t+1} = K - 98$.

since $\dfrac{PR_{t+1}}{PR_t} = 1+i = 1+8\%$. then $(K-98) = 1.08(K-103) \Rightarrow K = 165.50$.

$PR_{t+1} = 165.50 - 98 = 67.50$

(b) set n as the duration of repayment of Feb. 2003

(t as Feb 2004, $t-1$ as Feb 2003)

$OB_{t-1} = \dfrac{I_t}{i} = \dfrac{103}{0.08} = 1287.50$.

Set OB_{t-1} as initial value, then

$OB_{t-1} = K\, a_{\overline{n|}0.08} \Rightarrow 1287.50 = 165.50\, a_{\overline{n|}0.08} \Rightarrow n = 12.7$,

which means 12 regular payments and a smaller final payment X,

Then

$1287.50 = 165.50\, a_{\overline{12|}i} + X \cdot v_i^{13} \Rightarrow X = 109.54$.

The date of final payment is Feb, 2016.

3.2-9.

Suppose she gets the payment from bank A is 1 per month.

$i = \dfrac{i^{(12)}}{12} = 0.01$,

The repayment starts at the end of 5 yr.

$1 \cdot s_{\overline{120|}} = \dfrac{(1.01)^{120} - 1}{0.01}$

$OB_{5\times 12} = OB_{60} = 1 \cdot a_{\overline{180-60|}0.01} = a_{\overline{120|}0.01} = 69.7$ (it has been

Plus the penalty, $L = OB_{60} \times (1+K\%) = 69.7(1+K\%)$ is loan from Bank B.

Assume repayment is R, consider the time point at the end of 5^{th} year

$R \cdot a_{\overline{120|}0.0075} = L \Rightarrow R = 0.883 \cdot (1+K\%)$.

correct it $\quad R \leq 1 \Rightarrow 0.883(1+K\%) \leq 1 \Rightarrow K \leq 0.1326$

3.7.12.

At $i^{(12)} = 0.12$. the monthly payment $K = \frac{100,000}{a_{\overline{25\times12}|0.01}} = 1053.22$.

after 3 years. Outstanding balance $OB_{12\times3} = 1053.22\, a_{\overline{300-36}|0.01} = 97.707.45$.

Jones still pay the same amount of money.

~~$OB_{12\times3}$~~ a.

$97707.45\, v_{0.0125}^{36} + 1053.22\, a_{\overline{36}|0.0125} = 92,858$.

Smith receives 192, 858 in total.

Tutorial: 3.2.6, 3.2.9, 3.2.12 Problem Set: ~~[illegible]~~, 3.2.7, 3.2.8, 3.2.10, 3.2.11, 3.2.16.

3.2.7.

$X(1.06)^{10} - X$ is the total interest paid at the end of 10^{th} year.

$\frac{X}{a_{\overline{10}|6\%}}$ is the payment for each level

$$\left(X(1.06)^{10} - X\right) - \left(10 \cdot \frac{X}{a_{\overline{10}|6\%}} - X\right) = 356.54 \Rightarrow X = 825.$$

3.2.8.

(i) $K = \frac{2000}{a_{\overline{10}|8.07\%}} = 299 \Rightarrow 10K = 2990$

(ii) $200 + 2000i,\ 200 + 1800i, \ldots, 200 + 200i,$

total $2000 + 200i(10 + 9 + \cdots + 1) = 2000 + 200i \cdot \frac{10 \times 11}{2} = 2990 \Rightarrow i = 9\%.$

3.2.10.

(a)

Present value of interest: $\overset{I_1}{\overset{3.1}{10}v} + \overset{I_2}{8.99}v^2 + \cdots + \overset{I_6}{7.29}v^6 = 39.33$

Present value of principal: $\overset{PR_1}{\overset{3.1}{105.61}v} + \overset{PR_2}{106.67}v^2 + \cdots + \overset{PR_6}{228.91}v^6 = 960.67$

interest PV 3.4 : $5(Da)_{\overline{n}|r_2} = 356.16.$

principal PV 3.4 : $250\, a_{\overline{n}|} = 2643.84$

(b)

payment for each level is $K = \frac{L}{a_{\overline{n}|}}$, $I_t = K(1 - v^{n-t+1})$, $PR_t = Kv^{n-t+1}$

PV of the interest: $\sum_{t=1}^{n} K(1 - v^{n-t+1}) \cdot v^t = L\left[1 - \frac{nv^{n+1}}{a_{\overline{n}|}}\right].$

PV of the principal: $\sum_{t=1}^{n} Kv^{n-t+1} \cdot v^t = L \cdot \frac{nv^{n+1}}{a_{\overline{n}|}}$

3.2.11.

$OB_t = 5190.72$, $OB_{t+1} = 5084.68$, $OB_{t+2} = 4973.66.$

$PR_{t+1} = OB_t - OB_{t+1} = 106.04,$ $PR_{t+2} = OB_{t+1} - OB_{t+2} = 111.02.$

$\Rightarrow 1 + j = \frac{PR_{t+2}}{PR_{t+1}} = 1.047.$ $I_{t+1} = OB_t \times i = 5190.72 \times 4.7\% = 243.77$

$K = I_{t+1} + PR_{t+1} = 349.81.$

3.2-16.

(a) (i) $K = \frac{L}{a_{\overline{n}|i}} = \frac{Li}{1-v_i^{\,n}}$. annual payment

(ii) $J = \frac{L}{a_{\overline{12n}|j}} = \frac{Lj}{1-v_j^{\,12n}}$, monthly payment

since $(1+j)^{12} = 1+i \Rightarrow v_j^{\,12n} = v_i^{\,n}$,

for (i) $OB_t = K \cdot a_{\overline{n-t}|i} = L \cdot \frac{a_{\overline{n-t}|i}}{a_{\overline{n}|i}} = L \cdot \frac{1-v_i^{\,n-t}}{1-v_i^{\,n}}$

for (ii) $OB_t = J \cdot a_{\overline{12n-t}|j} = L \frac{1-v_i^{\,12(n-t)}}{1-v_j^{\,12n}} = L \cdot \frac{1-v_i^{\,n-t}}{1-v_i^{\,n}}$ same as (i).

(b)

for (i) $I_T^{(i)} = \sum I_t = \sum (OB_{t-1} \times i) = i \sum (OB_0 + OB_1 + \cdots + OB_{n-1})$

for (ii) $I_T^{(ii)} = j \sum (OB_0' + OB_1' + \cdots + OB_{12n-1}')$.

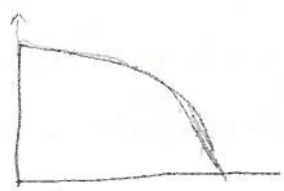
since $\cancel{j}$ $j = \frac{i^{(12)}}{12} < \frac{i}{12} \Rightarrow i \geq 12j$.

$I_T^{(i)} = 12j \sum (OB_0 + OB_1 + \cdots + OB_{n-1})$

since $0 + OB_1' + \cdots + OB_n' + OB_n' > 12 OB_1$

$(OB_1' + \cdots + OB_n')j > 12 OB_1 j$.

$I_T^{(ii)}$



$I_T^{(i)} = \sum_{t=1}^{n} I_t = \sum_{t=1}^{n} K(1-v_i^{\,n-t+1}) = \sum_{t=1}^{n} \frac{Li}{1-v_i^{\,n}}(1-v_i^{\,n-t+1})$

$I_T^{(ii)} = \sum_{t=1}^{12n} I_t = \sum_{t=1}^{12n} J(1-v_j^{\,12n-t+1}) = \sum_{t=1}^{12n} \frac{Lj}{1-v_j^{\,12n}}(1-v_j^{\,12n-t+1})$